Dynamical properties of random charge fluctuations in a dusty plasma with different charging mechanisms

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A dust particle in a plasma acquires electric charge by collecting electrons and ions, and sometimes by emitting electrons. The charging currents consist of discrete charges, causing the charge to fluctuate around an equilibrium value. We developed a model yielding a general expression for the charge fluctuations' temporal autocorrelation function. Both the magnitude and characteristic time of fluctuations can be obtained, knowing the specific form of charging currents. Numerical results are presented for different charging mechanisms, including charging by thermionic and photoelectric emission. It is shown that for all charging mechanisms the amplitude of fluctuations varies as $\Delta Z = \alpha\sqrt{\langle |Z| \rangle}$, where $\langle Z \rangle$ is the equilibrium dust grain charge in units of electron charges. [S1063-651X(99)09004-2]

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I. INTRODUCTION

A dust particle immersed in a plasma acquires electric charge by collecting electrons and ions from plasma, and sometimes by emitting electrons. The most important emission processes are thermionic emission, secondary electron emission, and photoelectric emission [1]. When an equilibrium charge has been attained by the dust grains, a dusty plasma often can be regarded simply as a multispecie plasma with a constant grain charge. This treatment is useful in some cases. In general, however, an important distinctive feature of a dusty plasma is grain-charge fluctuation. Consequently, the dust electric charge becomes a time-dependent quantity and must be treated as a new dynamical variable.

The charge on a grain can fluctuate for two reasons. One cause is the turbulence or other spatial and temporal variations in the surrounding plasma properties (electron and ion temperatures and densities, current, etc.). Collective effects due to charge-fluctuation dynamics in a dusty plasma were investigated in Refs. [2,3]. Another cause of charge fluctuations is the discrete nature of the charge carriers. Electrons and ions are absorbed at or emitted by the grain surface at random times. For this reason, the charge fluctuates, even in a steady-state uniform plasma. The latter fluctuations, superimposed onto the equilibrium charge, are random, unlike the coherent fluctuations caused for the first reason.

The behavior of dust particles with fluctuating charges could be significantly different from what would be observed on the basis of the equilibrium charge alone. For example, fluctuations can alter the dust motion even in a plasma with constant electromagnetic fields, because both the electrostatic force and the Lorentz force will fluctuate as the charge fluctuates. In so-called dust crystals, charge fluctuations lead to a fluctuating intergrain potential that would have an effect similar to the random motion, in addition to the thermal one. For this reason, crystal formation might be inhibited to a certain degree by charge fluctuations. Including the emission processes can lead to charge fluctuations between positive and negative values. This would tend to promote grain growth by coagulation. The role of these effects depends on the magnitude and characteristic time of charge fluctuations, which must be determined.

Some studies appeared in recent years that addressed various aspects of charge fluctuations that arise from the random nature of the charging process in plasmas. Morfill, Grun, and Johnson [4] assumed that the number of charges residing on a dust grain varies randomly according to Poisson statistics, giving

$$\Delta Z = \sqrt{\langle |Z| \rangle}. \tag{1}$$

Goree and co-workers [1,5] developed a numerical simulation yielding a time series for the grain charge in a plasma in the absence of emission processes. The main result of their work is that the rms level of fluctuations is

$$\Delta Z = 0.5\sqrt{\langle |Z| \rangle} \tag{2}$$

for a wide range of plasma and grain parameters, provided that $\langle |Z| \rangle \gg 1$. Matsoukas and co-workers [6,7] presented an analytical model of stochastic charge fluctuations of dust particles surrounded by a stationary undisturbed plasma. This model can be used to quantify the static properties of charge fluctuations, e.g., charge distribution and the amplitude of fluctuations. Results were obtained for both Maxwellian and non-Maxwellian electrons as well as for charging by an ion wind [7], but emission processes were also excluded.

In this paper a model which involves the discrete nature of the charge carriers is developed to study random charge fluctuations, particularly their dynamical behavior. We develop a general expression for the charge fluctuations' temporal autocorrelation function (TAF) providing both the magnitude and characteristic time of fluctuations (the TAF, for example, can be very useful in numerical modeling of dusty plasma with fluctuating particle charge). Some analytical results for different charging conditions (including emission processes) are presented. Results of our model are compared with numerical simulations [5], and with a theoretical model [7] in the situation when emission processes are unimportant.

II. MODEL DESCRIPTION

We consider a particle with an instantaneous charge Z. Two assumptions will be used further for simplicity: the first is that all negative charges are carried by electrons; the second is that all positive ions (if they exist) are singly charged. Under these assumptions the charge dynamics equation is given by

$$\frac{dZ}{dt}=I=\sum I^{+}-I^{-}, \tag{3}$$

where ΣI^{+} is the sum of all currents charging a dust particle positively (a collection of ions from the plasma and electron emission currents), and I^{-} is the current charging a dust particle negatively (a collection of electrons from the plasma). The steady-state (equilibrium) charge $\langle Z\rangle$ is determined by equating the right hand side of Eq. (3) to zero, so that opposing currents are balanced. Due to charge fluctuations, the charge of a dust grain will be a time-dependent variable. Considering small fluctuations from the equilibrium charge, we introduce $Z(t)=\langle Z\rangle+\delta Z(t)$, where $|\langle Z\rangle| \gg |\delta Z(t)|$, and rewrite Eq. (3) in the form (retaining terms up to first order)

$$\frac{d\delta Z}{dt}=\left[\left.\frac{\partial I}{\partial Z}\right|_{z=\langle z\rangle}\right]\delta Z=-\beta\delta Z, \tag{4}$$

where the term $\beta\delta Z$ ($\beta>0$) acts as a restoring force that tends to bring the charge to the equilibrium value determined by the condition $I=0$. The charging current was treated here as if it was continuous in time. This continuous charging model provides information about the equilibrium charge and decay rate of small deviations of a grain charge from its equilibrium. We will now extend the continuous charging model to include the effect of discrete charges.

Let us consider the collection or emission of a single discrete charge (electron or ion). The sequences in which electrons and ions arrive at the grain surface, or electrons are emitted by the surface, are random. The time intervals between the successive acts of absorption or emission are also random variables. That is why the grain charge can fluctuate. To describe this process, we rewrite the charge dynamics equation (4), introducing a term. This term is responsible for the fluctuations in both directions around the equilibrium charge,

$$\frac{d\delta Z}{dt}+\beta\delta Z=F(t), \tag{5}$$

where $F(t)$ is the stochastic term given by

$$F(t)=\sum_{j}\delta(t-t_j)(\pm 1)_j, \tag{6}$$

corresponding to the absorption of plasma ion or electron or to the emission of an electron at random moment t_j. The $+$ sign corresponds to the positive change in the grain charge (emission of an electron or absorption of an ion), and the $-$ sign corresponds to the absorption of an electron from the plasma. In Eq. (5) the charging process is decomposed into a rapidly varying term $F(t)$ and a slowly varying term $\beta\delta Z$. This semiphenomenological treatment is valid because a great difference in time scales. Note that Eq. (5) is equivalent to the Langevin equation of motion of a free Brownian particle in one dimension if we treat δZ as a particle velocity and $F(t)$ as a random force arising from the atom or molecular bombardment (Brownian force). The similarity of these two processes is obvious. The stochastic term $F(t)$ satisfies

$$\langle F(t)\rangle=0,\quad \langle F(t)F(t')\rangle=\frac{1}{t_0}\delta(t-t'). \tag{7}$$

Here $1/t_0$ characterizes the frequency of plasma particles (electrons and ions) absorption and emission ($1/t_0=\Sigma I^{+}+I^{-}=2I^{-}$). The solution of Eq. (5) can be written in the form

$$\delta Z(t)=\delta Z(0)\exp(-\beta t)+\exp(-\beta t)\int_0^t F(\theta)\exp(\beta\theta)d\theta. \tag{8}$$

Here $\delta Z(0)$ is the initial deviation of a grain charge from equilibrium. Assuming that $\delta Z(0)=0$, so that initially the charge is unperturbed, using Eq. (7) we found the temporal autocorrelation function of grain charge fluctuations:

$$\langle\delta Z(t)\delta Z(t')\rangle=\frac{1}{2t_0\beta}\exp(-\beta|t-t'|). \tag{9}$$

Equation (9) is the main result of this section. One can see from Eq. (9) that the correlation time of random charge fluctuations is $1/\beta$. The rms level (amplitude) of fluctuations can easily be found by substituting $t=t'$ in Eq. (9), giving

$$\Delta Z\equiv\sqrt{\langle\delta Z^2\rangle}=(2t_0\beta)^{-1/2}. \tag{10}$$

Note that Eq. (9) is obtained with no specific reference to the analytical form of the charging currents, and is applicable to any charging process provided that the negative charges are carried by electrons and all positive ions are singly charged. The only restriction of the model described is that fluctuations are small compared to the equilibrium charge.

III. RESULTS

In this section some results are obtained using the model outlined in Sec. II. To do this, the simplest models of particle charging are considered, including particle charging by thermionic and photoelectric emission. It is shown that for all charging mechanisms the amplitude of fluctuations can be represented in the form $\Delta Z=\alpha\sqrt{\langle|Z|\rangle}$. Thus the scaling is the same as in counting statistics where the uncertainty of a charge Z is $Z^{1/2}$. However, the coefficient α as well as characteristic fluctuation frequency β can be predicted using our model if charging currents are specified.

A. Charging by collection of ions and electrons from plasma

We consider a spherical particle of radius a, immersed in a steady-state uniform plasma with a shielding length λ. The emission processes are not included, and the particle is charged only by collecting electrons and ions from the plasma. Under the condition $a\ll\lambda\ll\lambda_{\mathrm{mfp}}$, where λ_{mfp} is a

TABLE I. The values of γ, α, and β (see text) for different plasma conditions (M_i and T_i/T_e)

M_i (amu)	T_i/T_e	γ	α	β (s^{-1})
1	0.05	1.698	0.61	1.1×10^5
1	1	2.501	0.56	4.0×10^4
40	0.05	2.989	0.50	2.5×10^4
40	1	3.952	0.46	8.7×10^3

mean free path for electron-neutral or ion-neutral collisions, we can use the ''orbital-motion-limited'' (OML) theory [8] to describe the currents collected by the grain. This simple theory is based on the conservation of angular momentum and the conservation of energy for collected electrons and ions, and the absence of an absorption radius (replacing the particle radius) for ions. The latter condition does not hold only in a relatively dense plasma [8]. In this paper we will not consider this situation. Assuming Maxwellian distributions both for electrons and ions and the vacuum capacitance of a dust particle in a plasma, for electron and ion currents we have

$$I^- = I_e = \pi a^2 n_e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} \exp\left(\frac{Ze^2}{aT_e}\right), \tag{11}$$

$$I^+ = I_i = \pi a^2 n_i \left(\frac{8T_i}{\pi m_i}\right)^{1/2} \left(1 - \frac{Ze^2}{aT_i}\right), \tag{12}$$

where $n_{e(i)}$, $m_{e(i)}$, and $T_{e(i)}$ are the electron (ion) concentration, mass, and temperature, respectively, e is the electron charge, and Z is assumed to be negative. If the dust particle concentration is sufficiently low so that $n_e \approx n_i$ for the equilibrium charge we can write [5] $\langle Z\rangle = -\gamma(T_i/T_e, m_i/m_e)(aT_e/e^2)$, where $\gamma > 0$ is a coefficient (of the order of unity) depending on plasma parameters. This coefficient can be determined numerically [5]. Using Eqs. (11) and (12), one can find

$$\beta = -\left.\frac{\partial I}{\partial Z}\right|_{z=\langle z\rangle} = \pi a^2 n_e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} \frac{e^2}{aT_e} \times \left[\left(\frac{m_e}{m_i}\frac{T_e}{T_i}\right)^{1/2} + \exp(-\gamma)\right] \tag{13}$$

and

$$\Delta Z = \left[\frac{1+\gamma\theta}{\gamma(1+\theta+\gamma\theta)}\right]^{1/2} \sqrt{\langle |Z|\rangle} = \alpha\sqrt{\langle |Z|\rangle}, \tag{14}$$

where $\theta = T_e/T_i$. The same expression for the amplitude of fluctuations (for α) was obtained in Refs. [6,7]. In Ref. [5] it was found through computer simulations that this coefficient is 0.5 for a wide range of plasma and grain parameters. In Table I the values of α are listed as determined from Eq. (14) for plasma parameters used in simulations [5]. In addition, the values of β for 1-mcm-diameter particle, $n_e = 5\times10^8\,\mathrm{cm}^{-3}$ and $T_e = 4$ eV, are listed. It can be seen from Table I that α depends only weakly on plasma parameters, and all the values of α are close to 0.5. At the same time, the characteristic fluctuation frequency β is strongly affected by change in plasma parameters.

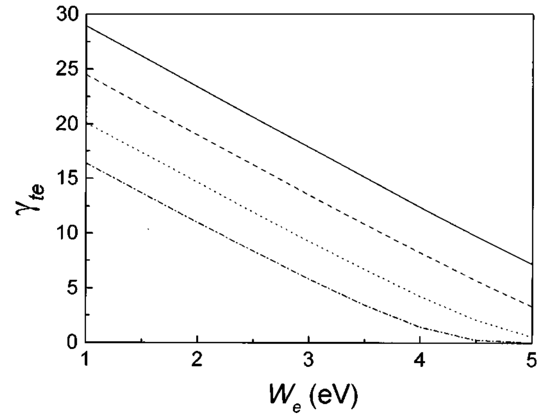

FIG. 1. Dependence of γ_{te} on the work function W_e for a particle charged by thermionic emission (the solid line corresponds to $n_p = 10^2$, the dashed line to $n_p = 10^4$, the dotted line to $n_p = 10^6$, and the dash-dotted line to $n_p = 5\times10^7$). The plasma temperature is assumed to be 2000 K.

B. Charging by thermionic emission

Thermionic emission plays an important role in a thermal dusty plasma, used recently to study the formation of ordered structures of dust particles [9]. We consider the simplest system, consisting of positively charged dust particles and emitted electrons (there are no ions in this system). Charge neutrality requires that

$$n_e = \langle Z\rangle n_p. \tag{15}$$

In the case of a thermal dusty plasma, which is characterized by the equivalence of electron, gas, and particle temperatures, electron flow from the particle surface can be given by [10]

$$I_{te}^+ = 2\pi a^2 \left(\frac{m_e T_e}{2\pi\hbar^2}\right)^{3/2} \left(\frac{8T_e}{\pi m_e}\right)^{1/2} \left(1 + \frac{Ze^2}{aT_e}\right) \times \exp\left(-\frac{W_e}{T_e} - \frac{Ze^2}{aT_e}\right), \tag{16}$$

where W_e is the work function. The expressions for the electron collection current (obtained using OML theory) is

$$I^- = \pi a^2 n_e \left(\frac{8T_e}{\pi m_e}\right)^{1/2} \left[1 + \frac{Ze^2}{aT_e}\right]. \tag{17}$$

The equilibrium average charge is maintained by the balance between electron recombination current and emission current. This charge can be written (similarly to the previous case) in the form $\langle Z\rangle = \gamma_{te}(aT_e/e^2)$, where $\gamma_{te} = \gamma_{te}(W_e, n_p)$ is a parameter which can be determined numerically. The value of this parameter as a function of W_e for different particle concentrations is shown in Fig. 1 [the gas temperature is assumed to be $T_e = 0.17$ eV (2000 K)]. It

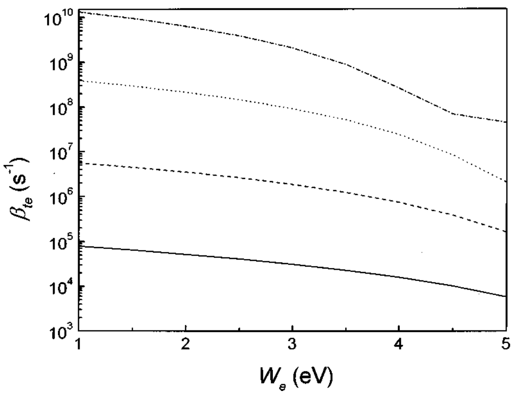

FIG. 2. Dependence of β_{te} on the work function W_e for a particle charged by thermionic emission (the solid line corresponds to $n_p=10^2$, the dashed line to $n_p=10^4$, the dotted line to $n_p=10^6$, and the dash-dotted line to $n_p=5\times10^7$). The plasma temperature is assumed to be 2000 K, and the particle radius is $a=1$ mcm.

can be seen that the dependence of γ_{te} on W_e is practically linear for not very high dust concentrations.

Using Eqs. (4), (16), and (17) for the fluctuation frequency we obtain the following expression:

$$\beta_{te}=\pi a^2 n_p\left(\frac{8T_e}{\pi m_e}\right)^{1/2}(1+\gamma_{te})^2. \tag{18}$$

For the amplitude of fluctuations we obtain, similarly to Eq. (14), $\Delta Z=\alpha_{te}\sqrt{\langle Z\rangle}$, where

$$\alpha_{te}=(1+\gamma_{te})^{-1/2}. \tag{19}$$

The dependences of β_{te} and α_{te} on the work function W_e are shown in Figs. 2 and 3 (we have used $a=1$ mcm to determine β_{te}, and $T_e=0.17$ eV). Note that the characteristic fluctuation frequency can be very high compared to the situation when the particle is charged only by ion and electron absorption (see Table I). For a higher work function W_e, α_{te} tends to unity as $\gamma_{te}\leqslant 1$.

We will use conditions of the experiment in Ref. [9] for numerical evaluations, e.g., particle radius $a=0.4$ mcm, $n_p=5\times10^7$ cm^{-3}, $T_e=0.15$ eV, and $W_e=2.1$ eV. The charge

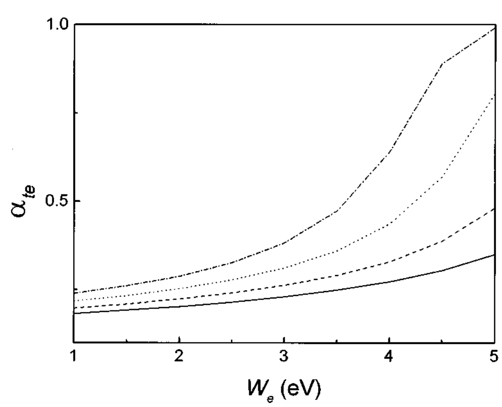

FIG. 3. Dependence of α_{te} on the work function W_e for a particle charged by thermionic emission (the solid line corresponds to $n_p=10^2$, the dashed line to $n_p=10^4$, the dotted line to $n_p=10^6$, and the dash-dotted line to $n_p=5\times10^7$). The plasma temperature is assumed to be 2000 K.

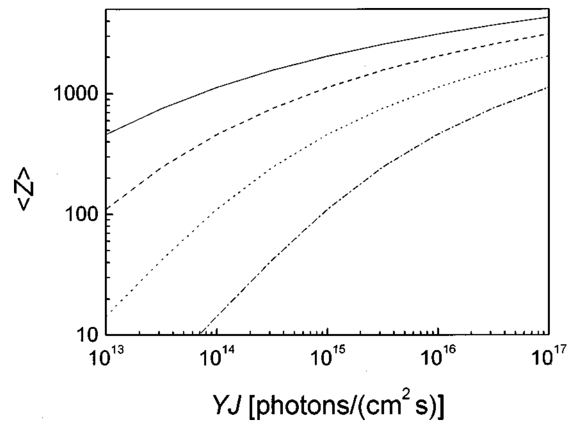

FIG. 4. Dependence of the equilibrium particle charge $\langle Z\rangle$ on parameter YJ for different dust concentrations (the solid line corresponds to $n_p=10^2$, the dashed line to $n_p=10^3$, the dotted line to $n_p=10^4$, and the dash-dotted line to $n_p=10^5$). The parameters used are $a=1$ mcm and $T_e\approx T_{pe}=1$ eV.

determined from the current balance [11] is $\langle Z\rangle=550$, in good agreement with the charge-determined experimentally from plasma quasineutrality condition [9]. So, we have $\gamma_{te}=13.2$ (see also Fig. 1), $\beta_{te}=1.2\times10^9$ s^{-1}, and $\alpha_{te}=0.28$.

C. Charging by UV irradiation

Charging by ultraviolet induced photoemission is a common process for a space plasma, and conditions for forming a Coulomb lattice of such charged dust grains in a laboratory have recently been extensively investigated [12,13]. Photoemission current from the grain is given by [13]

$$I_{pe}^{+}=\pi a^2 YJ\exp\left(-\frac{Ze^2}{aT_{pe}}\right), \tag{20}$$

where J is the UV photon flux, Y is the yield of the photoelectrons, and T_{pe} is their average energy. The electron collection current is given by Eq. (17). Taking into account the quasineutrality condition (15), from the current balance we obtain $\gamma_{pe}=\gamma_{pe}(n_p,a,YJ,T_e,T_{pe})$. For fluctuation characteristics we have, finally

$$\beta_{pe}=\pi a^2 n_p\left(\frac{8T_e}{\pi m_e}\right)^{1/2}\left[1+2\gamma_{pe}+\frac{T_e}{T_{pe}}\gamma_{pe}(1+\gamma_{pe})\right] \tag{21}$$

and

$$\alpha_{pe}=\left[\frac{1+\gamma_{pe}}{1+2\gamma_{pe}+(T_e/T_{pe})\gamma_{pe}(1+\gamma_{pe})}\right]^{1/2}. \tag{22}$$

If the neutral gas pressure in a system of particles charged by UV irradiation is not too high, electrons recombine on the grains faster than they can collide with neutrals and lose energy to heating the gas [12]. If this condition is satisfied we can assume that $T_e\approx T_{pe}$. In so doing, Eq. (22) is reduced to $\alpha_{pe}=[(1+\gamma_{pe})/(1+3\gamma_{pe}+\gamma_{pe}^2)]^{1/2}$.

For numerical illustration we assume a particle with $a=1$ mcm and electron temperature $T_e\approx T_{pe}=1$ eV. The equilibrium charge $\langle Z\rangle$ as a function of parameter YJ (characterizing the source of UV and dust particle material) is

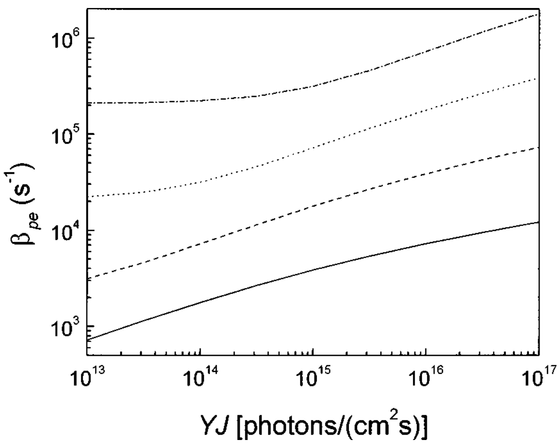

FIG. 5. Fluctuation frequency β_{pe} as a function of YJ for different particle concentrations. All parameters and notation follow Fig. 4.

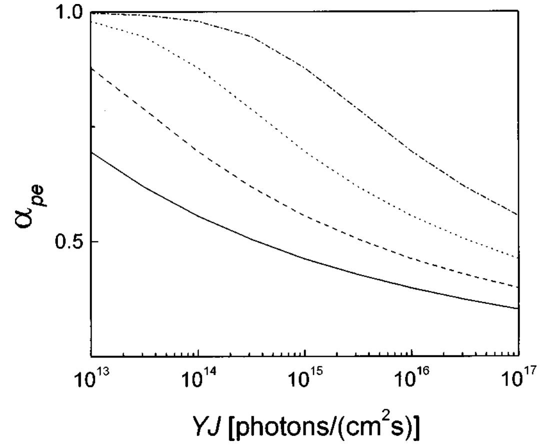

FIG. 6. Dependence of the fluctuation amplitude α_{pe} on YJ for different particle concentrations. All parameters and notation follow Fig. 4.

shown in Fig. 4 for different values of particle concentrations. We see that an increase in the dust density leads to a decrease in the particle charge due to an increase in the electron collection, while the photoemission current remains constant. Figures 5 and 6 show the dependence of the characteristic fluctuation frequency (β_{pe}) and fluctuation amplitude (α_{pe}) on parameter YJ. For high values of the dust density ($\gamma_{pe}\rightarrow 0$, according to Fig. 4), $\alpha_{pe}\rightarrow 1$. Dust densities used in the calculations of this section are smaller than in Sec. III B, because the optical depth of the system should be small to provide uniform charging.

IV. SUMMARY

Stochastic charge fluctuations may have important consequences on dust particle dynamics in a plasma. To evaluate the magnitude of the effect introduced by these fluctuations, a knowledge of their characteristics is necessary. In this paper a model is developed which provides the most useful characteristic of random grain charge fluctuations—the temporal autocorrelation function. The difference between the model presented here and the model developed in Ref. [6] is that the latter is more convenient to obtain information on static properties of charge fluctuations—their statistical distribution. On the other hand, the theory presented here provides information both on the amplitude and the characteristic time of fluctuations.

The exponential form of the TAF,

$$\langle \delta Z(t)\,\delta Z(0)\rangle = \langle \delta Z^2\rangle \exp\left(-\frac{t}{\tau_c}\right), \tag{23}$$

is independent of the charging mechanism. The amplitude $\langle \delta Z^2\rangle$ and characteristic time τ_c of fluctuations can be found knowing the specific form of the charging currents and the dusty plasma parameters. The basic assumptions used are the following: fluctuations are small compared to the equilibrium charge, the negative charges are carried by electrons, and all positive ions (if they exist) are singly charged. In addition, we have considered isolated dust particles, so that charge fluctuations on different particles are uncorrelated. We suggest that this assumption is valid when the following condition is satisfied: $Ze^2 n_p^{1/3}/\min\{T_e,T_i\} \ll 1$. This means that the effective length of the interaction between charged dust particles and plasma electrons (ions) is much less than the interparticle distance.

Within the above restrictions, the theory presented here can be used to predict charge fluctuations in any system for which the charging currents are known. The quantitative results are presented as an example of charging by a collection of ions and electrons from the plasma (in the framework of the OML theory), by thermionic and photoelectric emission. It is shown that for all charging mechanisms the amplitude of the fluctuations can be written in the form $\Delta Z = \alpha\sqrt{\langle |Z|\rangle}$.

ACKNOWLEDGMENTS

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[1] J. Goree, Plasma Sources Sci. Technol. **3**, 400 (1994).
[2] M. R. Jana, A. Sen, and P. K. Kaw, Phys. Rev. E **48**, 3930 (1993).
[3] R. Varma, P. Shukla, and V. Krishan, Phys. Rev. E **47**, 3612 (1993).
[4] G. Morfill, E. Grun, and T. Johnson, Planet. Space Sci. **28**, 1087 (1980).
[5] C. Cui and J. Goree, IEEE Trans. Plasma Sci. **22**, 151 (1994).
[6] T. Matsoukas and M. Russell, IEEE Trans. Plasma Sci. **77**, 4285 (1995).
[7] T. Matsoukas, M. Russell, and M. Smith, J. Vac. Sci. Technol. A **14**, 624 (1996).
[8] J. Allen, Phys. Scr. **45**, 497 (1992).
[9] V. Fortov, A. Nefedov, O. Petrov, A. Samarian, and A.

Chernyshev, Phys. Rev. E **54**, R2236 (1996).
[10] M. Sodha and S. Guha, Adv. Plasma Phys. **4**, 219 (1971).
[11] Y. K. Khodataev, S. A. Khrapak, A. P. Nefedov, and O. F. Petrov, Phys. Rev. E **57**, 7086 (1998).
[12] M. Rosenberg and D. Mendis, IEEE Trans. Plasma Sci. **23**, 177 (1995).
[13] M. Rosenberg, D. Mendis, and D. Sheenan, IEEE Trans. Plasma Sci. **24**, 1422 (1996).